

# Properties of Potassium

T.G. Tiecke

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## 1 Introduction

This document is a stand-alone version of Appendix A of my PhD thesis [1]. It is meant to provide an overview of the properties of atomic potassium useful for experiments on ultracold gases. A thorough review of the properties of lithium has been given in the thesis of Michael Gehm [2, 3]. For the other alkali atoms extended reviews have been given for Na, Rb and Cs by Daniel Steck [4].

version	date	changes
1.0	2009	first standalone version
1.02	2011	fixed typos
1.03	2019	fixed units in captions of figure 1 and 2

## 2 General Properties

Potassium is an alkali-metal denoted by the chemical symbol K and atomic number  $Z = 19$ . It has been discovered in 1807 by deriving it from potassium hydroxide KOH. Being an alkali atom it has only one electron in the outermost shell and the charge of the nucleus is being shielded by the core electrons. This

Mass number $A$	Neutrons $N$	Abundance (%) [6]	$m$ (u) [8]	$\tau$ [9]	$I$ [9]
39	20	93.2581(44)	38.96370668(20)	stable	3/2
40	21	0.0117(1)	39.96399848(21)	$1.28 \times 10^9$ y	4
41	22	6.7302(44)	40.96182576(21)	stable	3/2

Table 1: Naturally occurring isotopes of potassium. The atomic number of potassium is  $Z = 19$ . The given properties are the atomic number  $A$ , the number of neutrons in the nucleus  $N$ , the abundance, the atomic mass  $m$ , the lifetime  $\tau$  and the nuclear spin  $I$ .

Melting point	$63.65^\circ\text{C}$ (336.8 K)	[10]
Boiling point	$774.0^\circ\text{C}$ (1047.15 K)	[10]
Density at 293 K	$0.862$ g/cm <sup>3</sup>	[10]
Ionization energy	$418.8$ kJ mol <sup>-1</sup>	[10]
	$4.34066345$ eV	[11]
Vapor pressure at 293 K	$1.3 \times 10^{-8}$ mbar	[5]
Electronic structure	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	

Table 2: General properties of potassium

makes the element very chemically reactive due to the relatively low ionization energy of the outermost electron. The basic physical properties of potassium are listed in Table 2. Potassium has a vapor pressure given in mbar by [5]:

$$\begin{aligned}
 \text{(solid)} \quad \log p &= 7.9667 - \frac{4646}{T} & 298 \text{ K} < T < T_m. \\
 \text{(liquid)} \quad \log p &= 7.4077 - \frac{4453}{T} & T_m < T < 600 \text{ K}
 \end{aligned} \tag{1}$$

Figure 1 depicts the vapor pressure over the valid range of Eq. 1.

Potassium has a chemical weight of 39.0983(1) [6] and appears naturally in three isotopes,  $^{39}\text{K}$ ,  $^{40}\text{K}$  and  $^{41}\text{K}$  which are listed in Table 1. The fermionic isotope  $^{40}\text{K}$  has two radioactive decay channels. In 89% of the cases it decays through a  $\beta^-$  decay of 1.311MeV resulting in the stable  $^{40}\text{Ar}$ . In the remaining 11% it decays through electron capture (K-capture) to  $^{40}\text{Ca}$  [7]. The former decay channel is commonly used for dating of rocks.

### 3 Optical properties

The strongest spectral lines of the ground state potassium atom are the D1 ( $^2S \rightarrow ^2P_{1/2}$ ) and D2 ( $^2S \rightarrow ^2P_{3/2}$ ) lines. The most recent high precision measurements of the optical transition frequencies of potassium have been published by Falke *et al.* [12]. Tables 3 to 8 list the properties of the D1 and D2 lines for the various isotopes.

The natural lifetime  $\tau$  of an excited state is related to the linewidth of the associated transition by

$$\Gamma = \frac{1}{\tau} \tag{2}$$

where  $\Gamma$  is the natural linewidth. A temperature can be related to this linewidth, which is referred to as the *Doppler temperature*

$$k_B T_D = \frac{\hbar \Gamma}{2}$$

where  $k_B$  is the Boltzmann constant. The wavenumber  $k$  and frequency  $\nu$  of a transition are related to the wavelength  $\lambda$  by

$$k = \frac{2\pi}{\lambda}, \quad \nu = \frac{c}{\lambda} \tag{3}$$

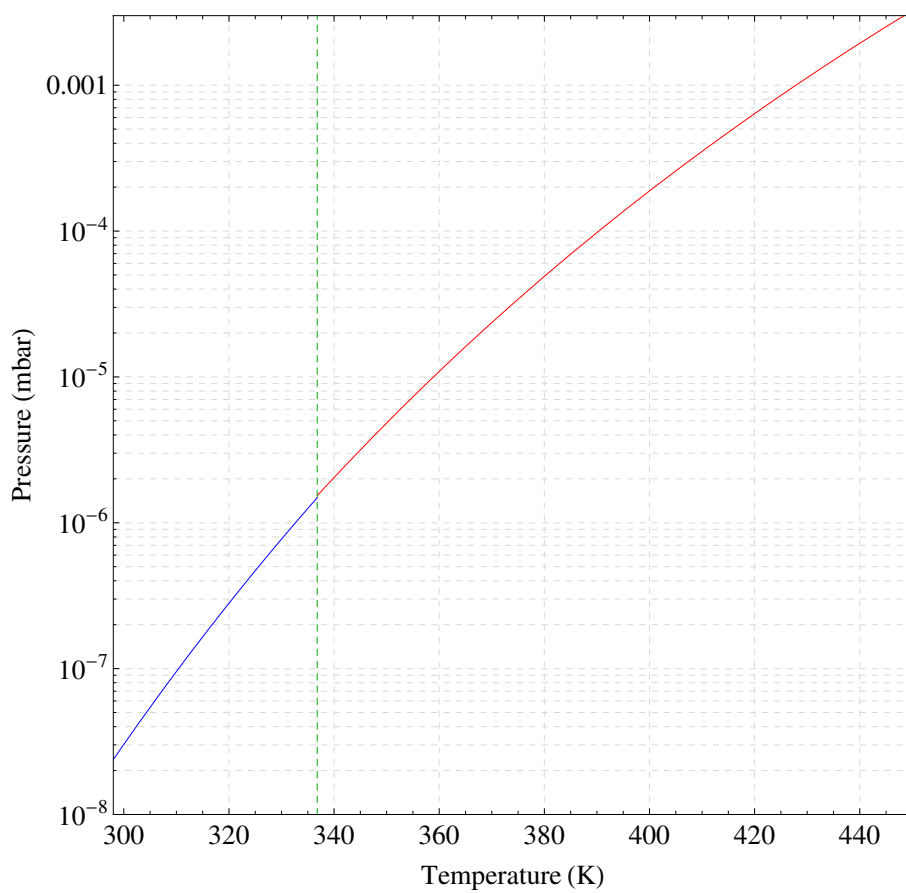


Figure 1: Vapor pressure of potassium taken from [5]. The green dashed line indicates the melting point of  $T = 336.8$  K.

Property	symbol	value	reference
Frequency	$\nu$	389.286058716(62) THz	[12]
Wavelength	$\lambda$	770.108385049(123) nm	
Wavenumber	$k/2\pi$	12985.1851928(21) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.72(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	5.956(11) MHz	
Recoil velocity	$v_{rec}$	1.329825973(7) cm/s	
Recoil Temperature	$T_{rec}$	0.41436702 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	

Table 3: Optical properties of the  $^{39}\text{K}$  D1-line.

Property	symbol	value	reference
Frequency	$\nu$	391.01617003(12) THz	[12]
Wavelength	$\lambda$	766.700921822(24) nm	
Wavenumber	$k/2\pi$	13042.8954964(4) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.37(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	6.035(11) MHz	
Recoil velocity	$v_{rec}$	1.335736144(7) cm/s	
Recoil Temperature	$T_{rec}$	0.41805837 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	
Saturation intensity	$I_s$	1.75 mW/ $\text{cm}^2$	

Table 4: Optical properties of the  $^{39}\text{K}$  D2-line.

When an atom emits or absorbs a photon the momentum of the photon is transferred to the atom by the simple relation

$$mv_{rec} = \hbar k \quad (4)$$

where  $m$  is the mass of the atom,  $v_{rec}$  is the recoil velocity obtained (lost) by the absorption (emission) process and  $\hbar = h/2\pi$  is the reduced Planck constant. A temperature can be associated to this velocity, which is referred to as the recoil temperature

$$k_B T_{rec} = \frac{1}{2} m v_{rec}^2 \quad (5)$$

Finally, we can define a saturation intensity for a transition. This intensity is defined as the intensity where the optical Rabi-frequency equals the spontaneous decay rate. The optical Rabi-frequency depends on the properties of the transition, here we only give the expression for a cycling transition

$$I_s = \frac{\pi \hbar c}{3\lambda^3 \tau}$$

Property	symbol	value	reference
Frequency	$\nu$	389.286184353(73) THz	[12]
Wavelength	$\lambda$	770.108136507(144) nm	
Wavenumber	$k/2\pi$	12985.1893857(24) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.72(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	5.956(11) MHz	
Recoil velocity	$v_{rec}$	1.296541083(7) cm/s	
Recoil Temperature	$T_{rec}$	0.40399576 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	

Table 5: Optical properties of the  $^{40}\text{K}$  D1-line.

Property	symbol	value	reference
Frequency	$\nu$	391.016296050(88) THz	[12]
Wavelength	$\lambda$	766.700674872(173) nm	
Wavenumber	$k/2\pi$	13042.8997000(29) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.37(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	6.035(11) MHz	
Recoil velocity	$v_{rec}$	1.302303324(7) cm/s	
Recoil Temperature	$T_{rec}$	0.40399576 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	
Saturation intensity	$I_s$	1.75 $\text{mW}/\text{cm}^2$	

Table 6: Optical properties of the  $^{40}\text{K}$  D2-line.

Property	symbol	value	reference
Frequency	$\nu$	389.286294205(62) THz	[12]
Wavelength	$\lambda$	770.107919192(123) nm	
Wavenumber	$k/2\pi$	12985.1930500(21) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.72(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	5.956(11) MHz	
Recoil velocity	$v_{rec}$	1.264957788(6) cm/s	
Recoil Temperature	$T_{rec}$	0.41408279 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	

Table 7: Optical properties of the  $^{41}\text{K}$  D1-line.

Property	symbol	value	reference
Frequency	$\nu$	391.01640621(12) THz	[12]
Wavelength	$\lambda$	766.70045870(2) nm	
Wavenumber	$k/2\pi$	13042.903375(1) $\text{cm}^{-1}$	
Lifetime	$\tau$	26.37(5) ns	[13]
Natural linewidth	$\Gamma/2\pi$	6.035(11) MHz	
Recoil velocity	$v_{rec}$	1.2070579662(7) cm/s	
Recoil Temperature	$T_{rec}$	0.41408279 $\mu\text{K}$	
Doppler Temperature	$T_D$	145 $\mu\text{K}$	
Saturation intensity	$I_s$	1.75 $\text{mW}/\text{cm}^2$	

Table 8: Optical properties of the  $^{41}\text{K}$  D2-line.

## 4 Fine structure, Hyperfine structure and the Zeeman effect

The fine structure interaction originates from the coupling of the orbital angular momentum  $\mathbf{L}$  of the valence electron and its spin  $\mathbf{S}$  with corresponding quantum numbers  $L$  and  $S$  respectively. The total electronic angular momentum is given by:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

and the quantum number  $J$  associated with the operator  $\mathbf{J}$  is in the range of  $|L - S| \leq J \leq L + S$ . The electronic ground state of  $^{40}\text{K}$  is the  $4^2S_{1/2}$  level, with  $L = 0$  and  $S = 1/2$ , therefore  $J = 1/2$ . For the first excited state  $L = 1$  and  $S = 1/2$  therefore  $J = 1/2$  or  $J = 3/2$  corresponding to the states  $4^2P_{1/2}$  and  $4^2P_{3/2}$  respectively. The fine structure interaction lifts the degeneracy of the  $4^2P_{1/2}$  and  $4^2P_{3/2}$  levels, splitting the spectral lines in the  $D_1$  line ( $4^2S_{1/2} \rightarrow 4^2P_{1/2}$ ) and the  $D_2$  line ( $4^2S_{1/2} \rightarrow 4^2P_{3/2}$ ).

The hyperfine interaction originates from the coupling of the nuclear spin  $\mathbf{I}$  with the total electronic angular momentum

$$\mathbf{F} = \mathbf{J} + \mathbf{I}$$

where the quantum number  $F$  associated with the operator  $\mathbf{F}$  is in the range of  $|J - I| \leq F \leq J + I$ , where  $I$  is the quantum number corresponding to the operator  $\mathbf{I}$ . For  $^{40}\text{K}$  the fine-structure splitting is  $\Delta E_{FS} \simeq h \times 1.7$  THz, therefore the two excited states can be considered separately when considering smaller perturbations like the hyperfine or Zeeman interaction which are on the order of a few GHz or less.

The Hamiltonian describing the hyperfine structure for the two excited states described above is given by [14, 15]

$$\mathbf{H}^{\text{hf}} = \frac{a_{\text{hf}}}{\hbar^2} \mathbf{I} \cdot \mathbf{J} + \frac{b_{\text{hf}}}{\hbar^2} \frac{3(\mathbf{I} \cdot \mathbf{J})^2 + \frac{3}{2}(\mathbf{I} \cdot \mathbf{J}) - \mathbf{I}^2 \mathbf{J}^2}{2I(2I - 1)J(2J - 1)},$$

where  $a_{\text{hf}}$  and  $b_{\text{hf}}$  are the magnetic dipole and electric quadrupole constants respectively. The dot product is given by

$$\mathbf{I} \cdot \mathbf{J} = \frac{1}{2}(\mathbf{F}^2 - \mathbf{I}^2 - \mathbf{J}^2)$$

This hyperfine interaction lifts the spin degeneracy due to the different values of the total angular momentum  $F$ . The energy shift of the manifolds are given by

$$\delta E_{\text{hf}} = \frac{a_{\text{hf}}}{2} [F(F + 1) - I(I + 1) - J(J + 1)]$$

For a  $S = 1/2$  system in the electronic groundstate,  $J = 1/2$ , the energy splitting due to the hyperfine interaction in zero field is given by

$$\Delta E_{\text{hf}} = \frac{a_{\text{hf}}}{2} \left( I + \frac{1}{2} \right)$$

In the presence of an external magnetic field the Zeeman interaction has to be taken into account

$$\mathbf{H}^Z = (\mu_B/\hbar)(g_J \mathbf{J} + g_I \mathbf{I}) \cdot \mathbf{B},$$

where  $g_J$  is the Landé g-factor of the electron and  $g_I$  the nuclear gyromagnetic factor. Note that different sign conventions for  $g_I$  are used in literature, here we take the convention consistent with the common references in this context [14, 4, 3], such that  $\mu = -g_I \mu_B \mathbf{I}$ . The factor  $g_J$  can be written as

$$g_J = g_L \frac{J(J + 1) - S(S + 1) + L(L + 1)}{2J(J + 1)} + g_S \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)},$$

where  $g_S$  is the electron g-factor,  $g_L$  is the gyromagnetic factor of the orbital, given by  $g_L = 1 - m_e/m_n$ , where  $m_e$  is the electron mass and  $m_n$  is the nuclear mass. The total hyperfine interaction in the presence of an external magnetic field is now given by the internal hamiltonian

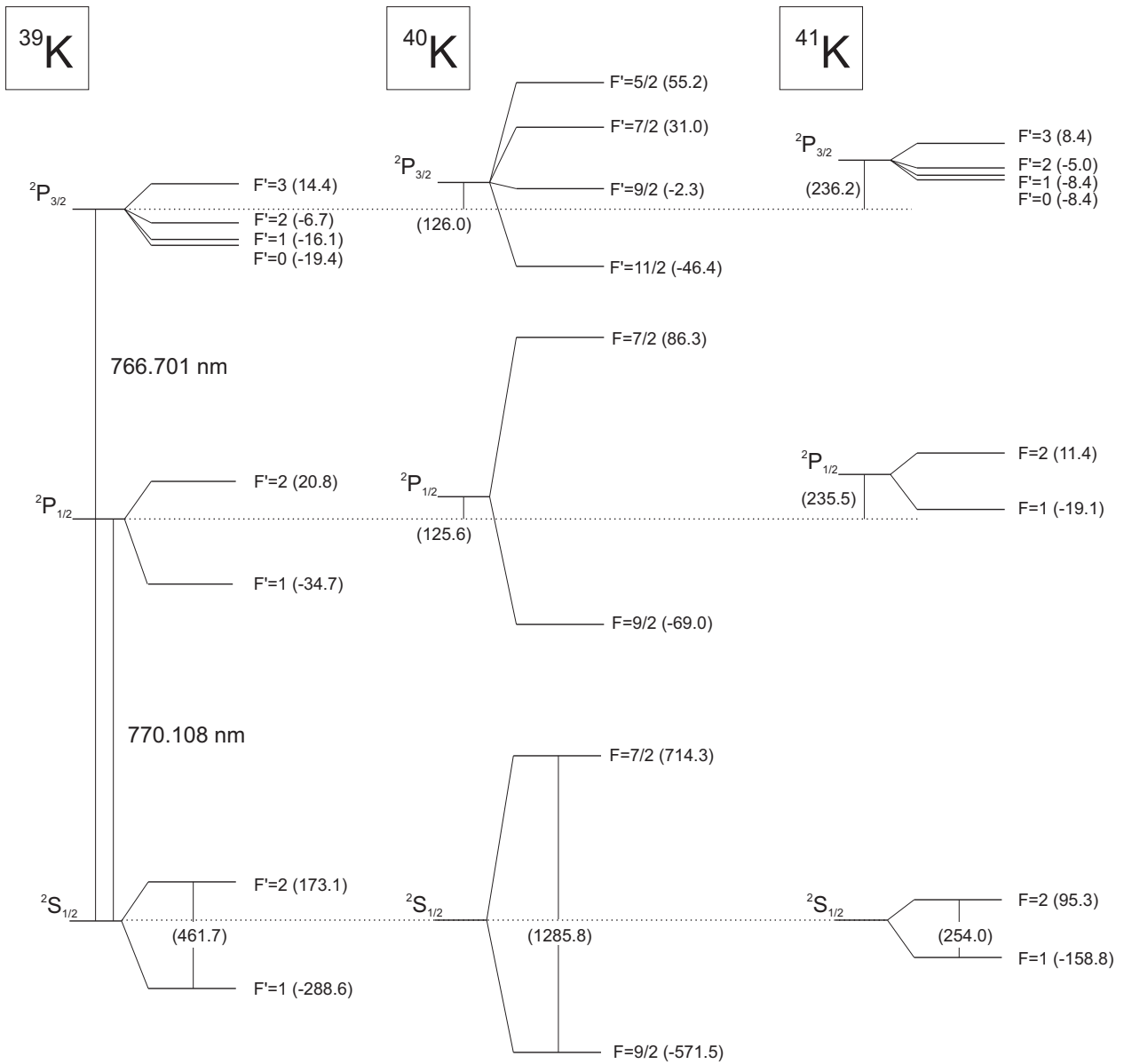


Figure 2: Optical transitions of the D1 and D2-lines of  $^{39}\text{K}$ ,  $^{40}\text{K}$  and  $^{41}\text{K}$ . A similar plot including  $^{37}\text{K}$ ,  $^{38}\text{K}$  can be found in [9]. The energy levels of the hyperfine structure are taken from [12] and [14] and given in units of MHz. Note the inverted hyperfine structure for  $^{40}\text{K}$ .

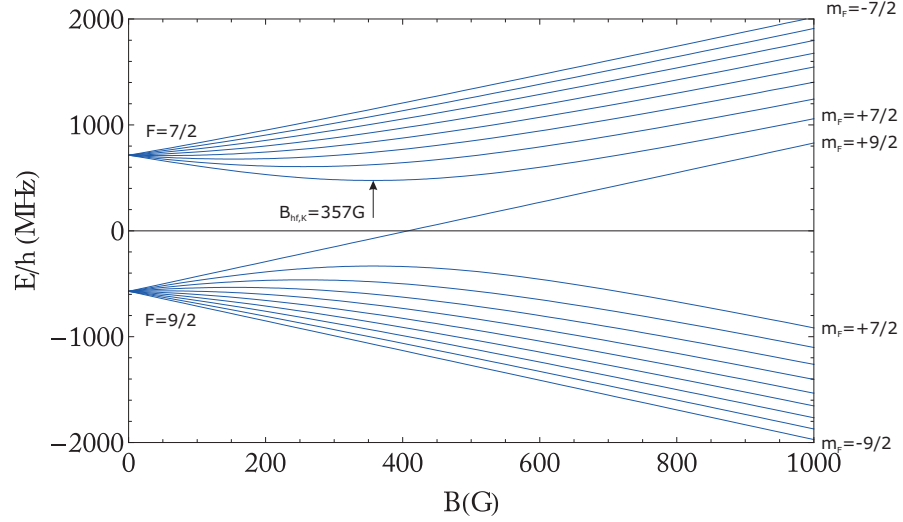


Figure 3: The hyperfine structure of the  ${}^2S_{1/2}$  groundstate of  ${}^{40}\text{K}$ . The states are labeled with their low-field quantum numbers  $|F, m_F\rangle$ . Note the inverted hyperfine structure.

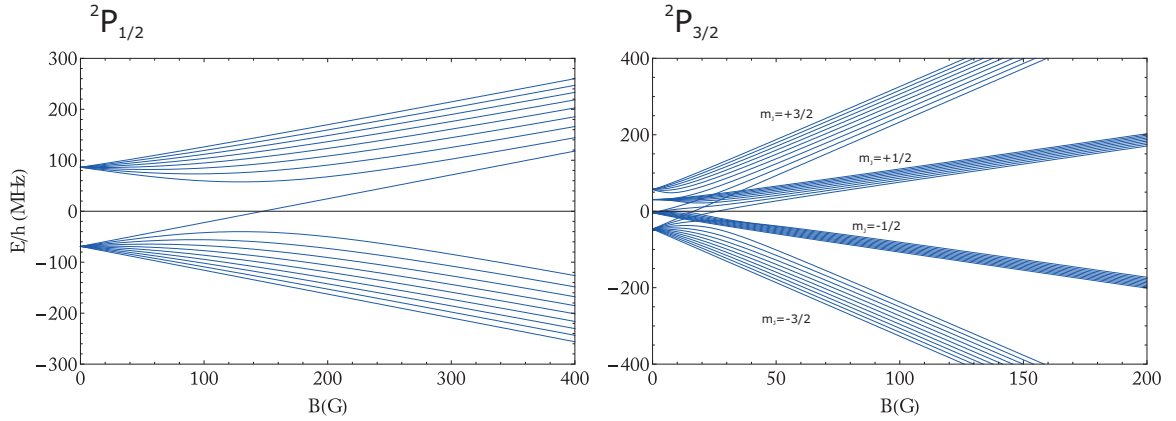


Figure 4: Hyperfine structure of the  ${}^2P_{1/2}$  (D1) and the  ${}^2P_{3/2}$  (D2) levels of  ${}^{40}\text{K}$ .

$$\mathbf{H}^{\text{int}} = \mathbf{H}^{\text{hf}} + \mathbf{H}^Z \quad (6)$$

In the absence of orbital angular momentum,  $L = 0$ , and for  $S = 1/2$ , the eigenvalues of Eq. 6 correspond to the *Breit-Rabi formula* [16]

$$E^{\text{hf}}(B) = -\frac{a^{\text{hf}}}{4} + g_I \mu_B m_f B \pm \frac{a^{\text{hf}}(I + 1/2)}{2} \left( 1 + \frac{4m_f x}{2I + 1} + x^2 \right)^{1/2} \quad (7)$$

$$x = \frac{(g_J - g_I) \mu_B}{a^{\text{hf}}(I + 1/2)} B$$

where  $\mu_B = 9.27400915 \times 10^{-24} \text{ JT}^{-1}$  is the Bohr-magneton and the sign corresponds to the manifolds with  $F = I \pm S$ .

Figures 3 and 4 show the eigenvalues of Eq. 6 for the  ${}^2S_{1/2}$  ground state and the  ${}^2P_{1/2}$  and  ${}^2P_{3/2}$  excited states of  ${}^{40}\text{K}$  respectively.



Constant	symbol	$^{39}\text{K } h \times \text{MHz}$	$^{40}\text{K } h \times \text{MHz}$	$^{41}\text{K } h \times \text{MHz}$	Ref.
$4^2S_{1/2}$ magnetic dipole	$a_{\text{hf}}$	230.8598601(3)	-285.7308(24)	127.0069352(6)	[14]
$4^2P_{1/2}$ magnetic dipole	$a_{\text{hf}}$	27.775(42)	-34.523(25)	15.245(42)	[12]
$4^2P_{3/2}$ magnetic dipole	$a_{\text{hf}}$	6.093(25)	-7.585(10)	3.363(25)	[12]
$4^2P_{3/2}$ electric quadrupole	$b_{\text{hf}}$	2.786(71)	-3.445(90)	3.351(71)	[12]

Table 9: Hyperfine structure coefficients for the ground state and the first excited state.

Property	Isotope	symbol	value	reference
Electron spin g-factor		$g_S$	2.0023193043622(15)	[17]
Total nuclear g-factor	$^{39}\text{K}$	$g_I$	-0.00014193489(12)	[14]
Total nuclear g-factor	$^{40}\text{K}$	$g_I$	+0.000176490(34)	[14]
Total nuclear g-factor	$^{41}\text{K}$	$g_I$	-0.00007790600(8)	[14]
Total electronic g-factor		$g_J(4^2S_{1/2})$	2.00229421(24)	[14]
Total electronic g-factor		$g_J(4^2P_{1/2})$	2/3	
Total electronic g-factor		$g_J(4^2P_{3/2})$	4/3	

Table 10: Electronic and nuclear gyromagnetic factors. Experimental values for the  $g_J$  values are not available, therefore, we use the Russel-Saunders values which agree within the error margins for all other alkali atoms [14].

#### 4.1 Transition strengths

In this section we present the transition strengths for  $^{40}\text{K}$ . We do not elaborate on the physics behind the transition dipole matrix elements. For a more thorough description and the transition strengths for  $^{39}\text{K}$  and  $^{41}\text{K}$  we refer to Ref. [18]. The transition matrix element coupling a ground state defined by the quantum numbers  $J, F, m_F$  to an excited state with quantum numbers  $J', F', m'_F$  is given by

$$\mu_{eg} = e \langle J' F' m'_F | \hat{\varepsilon} \cdot \mathbf{r} | J F m_F \rangle$$

where  $e$  is the electronic charge,  $\hat{\varepsilon}$  is the polarization unit vector of the optical electric field and  $\mathbf{r}$  is the position operator. The transition strength is proportional to the square of the matrix element  $D \sim |\mu_{eg}|^2$  and is given by

$$D \sim \left[ \sqrt{(2J+1)(2J'+1)(2F+1)(2F'+1)} \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix} \begin{Bmatrix} J' & F' & I \\ F & J & 1 \end{Bmatrix} \begin{pmatrix} F & 1 & F' \\ m_F & q & -m'_F \end{pmatrix} \right]^2$$

where the curly brackets denote the Wigner-6j symbol, the normal bracket the Wigner 3j-symbol and  $q = \pm 1$  for  $\sigma^\pm$  polarized transitions and  $q = 0$  for  $\pi$  transitions.

The relative strengths for the transitions  $D$  between different  $F, m_F$  values for  $^{40}\text{K}$  are shown in Fig. 5 and 6 for  $\sigma^+$  and  $\pi$  polarizations respectively. Transition strengths for  $\sigma^-$  can be obtained from Fig. 5 by replacing all  $m_F$  values with  $-m_F$ . The strengths are normalized to yield an integer value. Note that the normalizations are different for figures 5 and 6. Similar figures for  $^{39}\text{K}$  and  $^{41}\text{K}$  can be found in Ref. [18].

## 5 Scattering properties

The scattering properties of ultracold atoms are essential for the evaporative cooling processes and most experiments performed with ultracold gases. At typical densities temperatures the scattering reduces to  $s$ -wave scattering. For ultracold scattering only lower partial waves play a role and the scattering properties are determined by the positions of only the last few bound states of the potentials. The scattering can be described by the radial Schrödinger equation

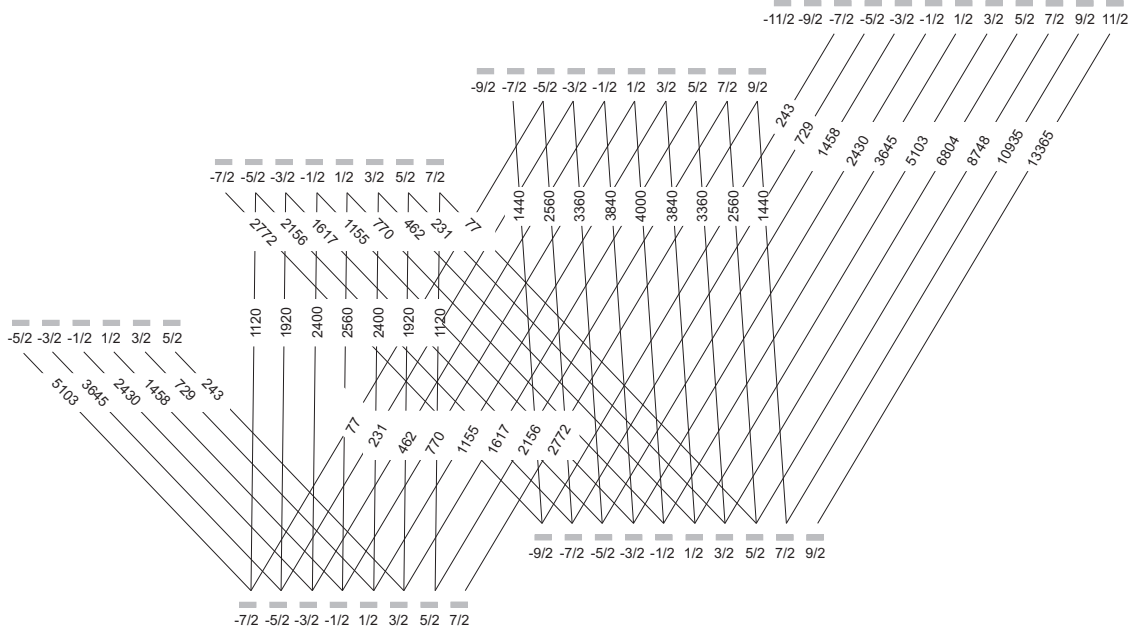


Figure 5: Transition probabilities for  $^{40}\text{K}$  ( $I = 4$ ) on  $\sigma^+$  transitions, normalized to integer values. Similar diagrams for  $^{39}\text{K}$  and  $^{41}\text{K}$  can be found in Ref. [18].

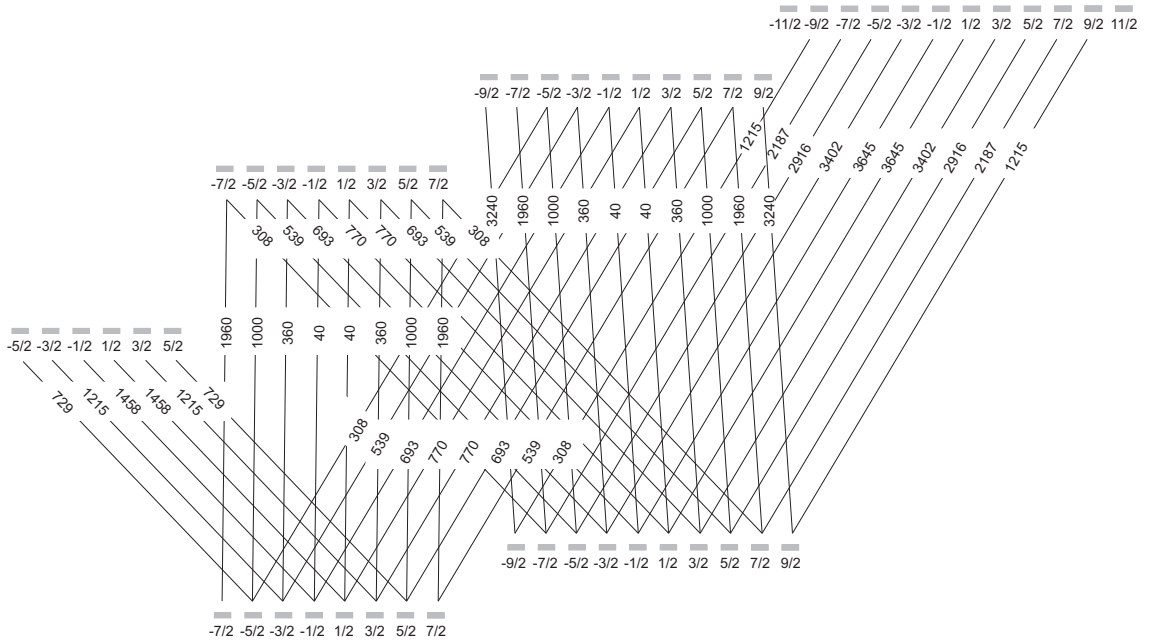


Figure 6: Transition probabilities for  $^{40}\text{K}$  ( $I = 4$ ) on  $\pi$  transitions, normalized to integer values. Similar diagrams for  $^{39}\text{K}$  and  $^{41}\text{K}$  can be found in Ref. [18].

isotope	$a_s$	$a_t$
39/39	138.49(12)	-33.48(18)
39/40	-2.84(10)	-1985(69)
39/41	113.07(12)	177.10(27)
40/40	104.41(9)	169.67(24)
40/41	-54.28(21)	97.39(9)
41/41	85.53(6)	60.54(6)

Table 11:  $s$ -wave scattering lengths for the various isotope-combinations of potassium, values are taken from Ref. [20]

	value	units
$C_6$	3925.9	$E_h a_0^6$
$C_8$	$4.224 \times 10^5$	$E_h a_0^8$
$C_{10}$	$4.938 \times 10^7$	$E_h a_0^{10}$
$r_0$ ( $^{39}\text{K}$ )	64.61	$a_0$
$r_0$ ( $^{40}\text{K}$ )	65.02	$a_0$
$r_0$ ( $^{41}\text{K}$ )	65.42	$a_0$

Table 12: Van der Waals properties of the scattering potential of potassium.  $V_{vdw}(r) = -C_6/r^6 - C_8/r^8 - C_{10}/r^{10}$ .

$$\left[ -\frac{\hbar^2}{2\mu} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \right) + V(r) \right] R(r) = \epsilon R(r), \quad (8)$$

where  $R(r)$  is the radial wavefunction,  $l$  is the angular momentum quantum number and  $V(r)$  is the scattering potential. Many ultracold scattering properties can be obtained with sufficient accuracy for general use in the lab by only using the accumulated phase method [19] and  $V(r) = -C_6/r^6$ . However, for Potassium accurate potentials have been published by Falke, *et al.* [20]. Because potassium has  $S = 1/2$  the total spin of the potassium dimer can be either singlet ( $S = 0$ ) or triplet ( $S = 1$ ). Figure 7 shows the Born-Oppenheimer potentials for the singlet  $X^1\Sigma$  and triplet  $a^3\Sigma$  potentials. Solving Eq. 8 for  $\epsilon \downarrow 0$  one can obtain the scattering length. Table 11 lists the  $s$ -wave scattering lengths of the various potassium isotopes [20].

To qualitatively describe the scattering for  $^{40}\text{K}$  we compare the scattering lengths to the the van der Waals range. The van der Waals range is a measure for the typical range of the potential for an atomic species. It is defined as the range where the kinetic energy of confinement in the potential equals the potential energy and is given by [21]

$$r_0 = \frac{1}{2} \left( \frac{2\mu C_6}{\hbar^2} \right)^{1/4}$$

Using the van der Waals coefficient of  $C_6 = 3925.9 E_h a_0^6$  [20] for  $^{40}\text{K}$  we obtain a van der Waals range of  $r_0 \simeq 65 a_0$ . The scattering lengths of both the singlet and triplet potentials are much larger than  $r_0$  indicating resonant scattering due to the presence of a weakly bound state in both the singlet and triplet scattering potentials. Figure 8 shows the wavefunctions of the least bound states in the singlet and triplet potentials for  $^{40}\text{K}$ . Note the horizontal logarithmic scale. The wavefunctions extend far into the asymptotic van der Waals tail of the potentials.

## 5.1 Feshbach resonances

The use of Feshbach resonances are essential for the study of ultracold gases, in particular for fermionic isotopes. A Feshbach resonance occurs due to a resonant coupling of a scattering pair of atoms with

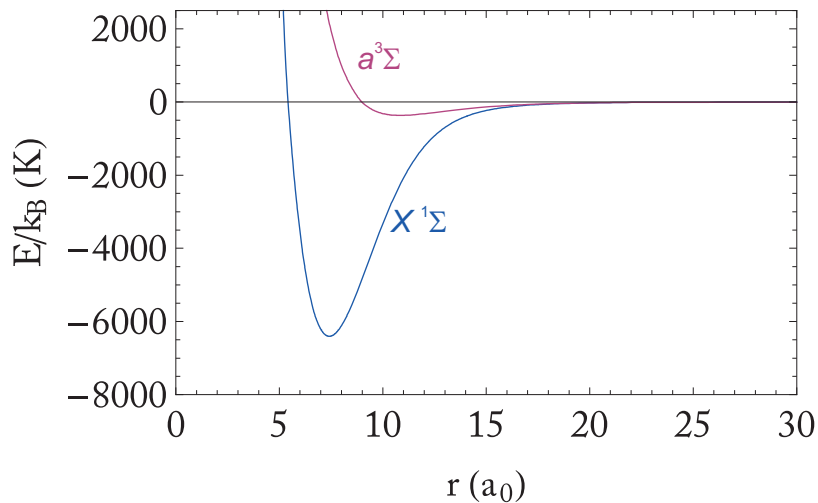


Figure 7: Born-Oppenheimer potentials  $V_S(r)$  for the singlet  $S = 0$ ,  $X^1\Sigma$  and the triplet  $S = 1$ ,  $a^3\Sigma$  states.

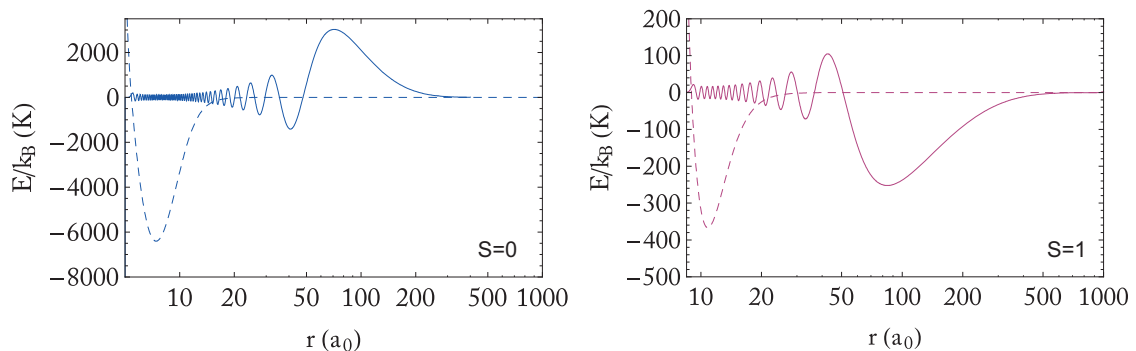


Figure 8: Wavefunctions of the least bound states of  $^{40}\text{K}$  for the singlet (left) and triplet (right) potentials. The dashed curve indicates the potential and the solid curve the radial wavefunction of the least-bound vibrational levels. Note the horizontal logarithmic scale and the asymptotic character of the wavefunctions.

$m_{f_1}, m_{f_2}$	$B_0$ (G)	$-\Delta B$ (G)	$a_{bg}$ ( $a_0$ )
1 + 1	$25.85 \pm 0.1$	0.47	-33
	$403.4 \pm 0.7$	52	-29
	(745.1)	0.4	-35
	$752.3 \pm 0.1$	0.4	-35
0 + 0	$59.3 \pm 0.6$	9.6	-18
	$66.0 \pm 0.9$	7.9	-18
	(471)	72	-28
	(490)	5	-28
	(825)	0.032	-36
	(832)	0.52	-36
-1 + -1	$32.6 \pm 1.5$	-55	-19
	$162.8 \pm 0.9$	37	-19
	$562.2 \pm 1.5$	56	-29

Table 13: Feshbach resonances for  $^{39}\text{K}$ . Numbers in brackets are only theoretically predicted. All values have been taken from Ref. [22]

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$m_{f_1}, m_{f_2}$	$s/p$	$B_0$ (G)	$\Delta B$ (G)	Ref.
$-9/2 + -7/2$	$s$	$202.10 \pm 0.07$	$7.8 \pm 0.6$	[23, 24, 25]
$-9/2 + -5/2$	$s$	$224.21 \pm 0.05$	$9.7 \pm 0.6$	[23, 26]
$-7/2 + -7/2$	$p$	$\sim 198.8$		[23, 24, 27]

Table 14: Feshbach resonances for  $^{40}\text{K}$ . All resonances are between spin states in the  $F = 9/2$  manifold. This table has been adapted from Ref. [23]

an energetically closed molecular state. The  $s$ -wave scattering length  $a$  in the vicinity of a Feshbach resonance is parameterized by

$$a(B) = a_{bg} \left( 1 - \frac{\Delta B}{B - B_0} \right)$$

where  $a_{bg}$  is the background scattering length in absence of coupling to the molecular state,  $B_0$  is the resonance position and  $\Delta B$  is the magnetic field width of the resonance. Due to the resonant scattering in the open channels (i.e. a large background scattering length) the Feshbach resonances of  $^{40}\text{K}$  have a broad character. For  $^{39}\text{K}$  eight resonances have been experimentally obtained and are listed together with some theoretical predictions in table 13. For  $^{40}\text{K}$  two experimentally characterized  $s$ -wave Feshbach resonances and one  $p$ -wave resonance have been published. The resonances are summarized in Table 14.

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